Digital Image Processing and Pattern Recognition

E1528

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Lecture 10

Image Restoration and Reconstruction

INSTRUCTOR

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➢ Introduction

- ➢ As in image enhancement, the principal goal of restoration techniques is to improve an image in some predefined sense. Although there are areas of overlap, image enhancement is largely a subjective process, while image restoration is for the most part an objective process.
- ➢ Restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon.
- ➢ Thus, restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image

➢ Objectives

- ➢ Be familiar with the characteristics of various noise models used in image processing, and how to estimate from image data the parameters that define those models.
- ➢ Be familiar with linear, nonlinear, and adaptive spatial filters used to restore (denoise) images that have been degraded only by noise.
- ➢ Know how to apply notch filtering in the frequency domain for removing periodic noise in an image.

➢ Objectives

- ➢ Understand the foundation of linear, space invariant system concepts, and how they can be applied in formulating image restoration solutions in the frequency domain.
- ➢ Be familiar with direct inverse filtering and its limitations.
- ➢ Understand minimum mean-square-error (Wiener) filtering and its advantages over direct inverse filtering.
- ➢ Understand constrained, least-squares filtering.
- ➢ Be familiar with the fundamentals of image reconstruction from projections, and their application to computed tomography.

➢ A Model of the Image Degradation/Restoration Process

- In this chapter, we model image degradation as an operator $\mathcal{H}$ that, together with an additive noise term, operates on an input image f(x,y) to produce a degraded image g(x,y).

- Given g(x,y), some knowledge about $\mathcal{H}$, and some knowledge about the additive noise term η(x,y), the objective of restoration is to obtain an estimate $\hat{f}$(x,y) of the original image.

- We want the estimate to be as close as possible to the original image and, in general, the more we know about $\mathcal{H}$ and η, the closer $\hat{f}$(x,y) will be to f(x,y).

➢ A Model of the Image Degradation/Restoration Process

- We will show that, if $\mathcal{H}$ is a linear, position-invariant operator, then the degraded image is given in the spatial domain by

$$g(x,y) = (h * f) + \eta(x,y)$$

- where h(x,y) is the spatial representation of the degradation function.
- the symbol "$*$" indicates convolution. It follows from the convolution theorem that the equivalent in the frequency domain is

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

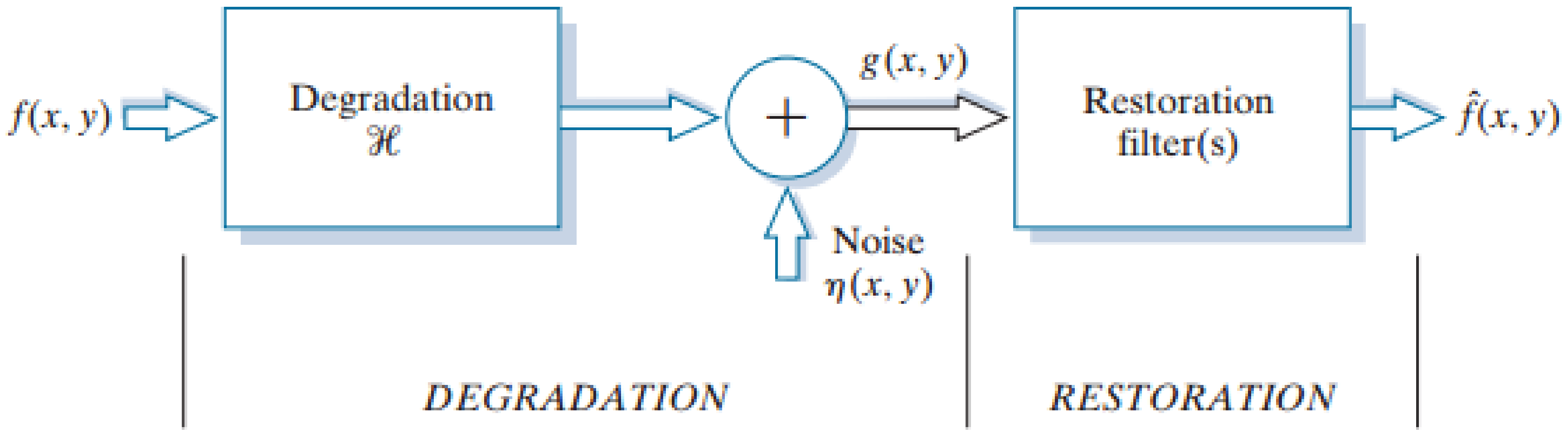
➢ Noise Models

- ➢ The principal sources of noise in digital images arise during image acquisition and/or transmission.
- ➢ The performance of imaging sensors is affected by a variety of environmental factors during image acquisition, and by the quality of the sensing elements themselves.
- ➢ For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image.

➢ Noise Models

➢ Images are corrupted during transmission principally by interference in the transmission channel.

➢ For example, an image transmitted using a wireless network might be corrupted by lightning or other atmospheric disturbance.



➢ Spatial and Frequency Properties of Noise

- Relevant to our discussion are parameters that define the spatial characteristics of noise, and whether the noise is correlated with the image.

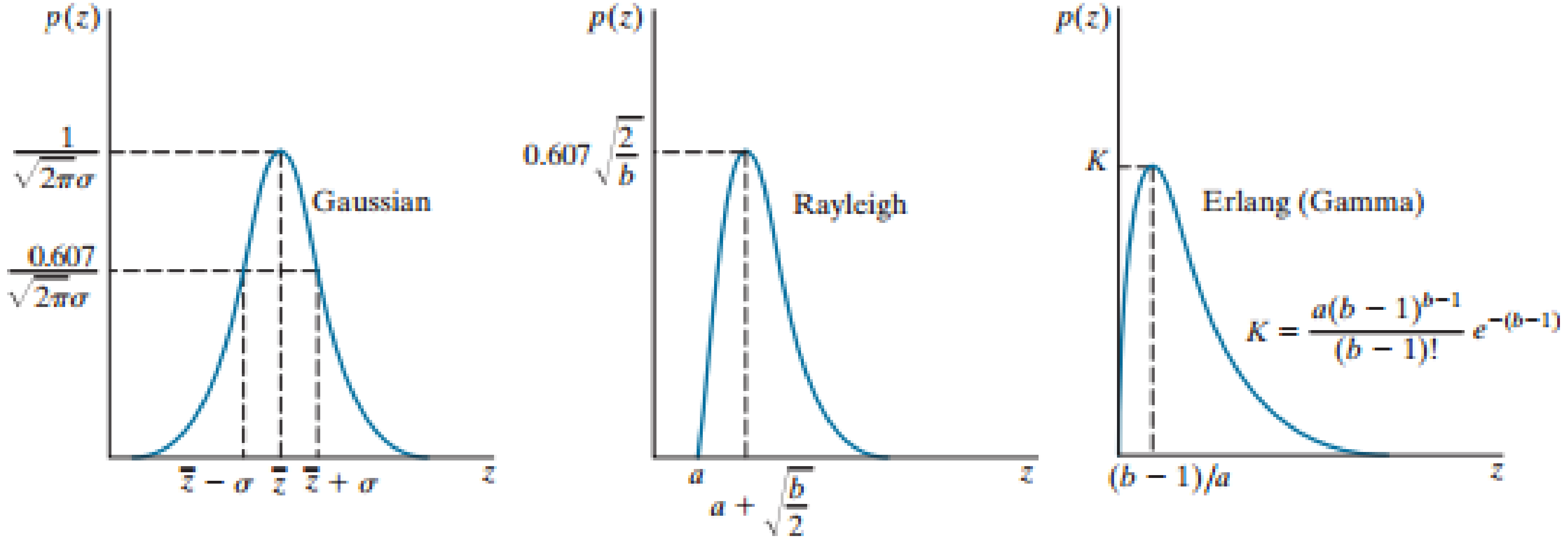
- Frequency properties refer to the frequency content of noise in the Fourier (frequency) domain

- when the Fourier spectrum of noise is constant, the noise is called white noise. This terminology is a carryover from the physical properties of white light, which contains all frequencies in the visible spectrum in equal proportions.

Some Important Noise Probability Density Functions

- We create a noise image for simulation purposes by generating an array whose intensity values are random numbers with a specified probability density function.
- This approach is true for all the PDFs to be discussed shortly, except for salt-and-pepper noise, which is applied differently.
- The following are among the most common noise PDFs found in image processing applications.

➢ Some Important Probability Density Functions.



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➢ Gaussian Noise

- Because of its mathematical tractability in both the spatial and frequency domains,
- Gaussian noise models are used frequently in practice. In fact, this tractability is so convenient that it often results in Gaussian models being used in situations in which they are marginally applicable at best.
- The PDF of a Gaussian random variable, z, is defined by the following familiar expression:

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\acute{z})^2}{2\sigma^2}} \qquad -\infty < z < \infty$$

➢ Gaussian Noise

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\acute{z})^2}{2\sigma^2}} \qquad -\infty < z < \infty$$

- ➢ where z represents intensity, ź is the mean (average) value of z, and σ is its standard deviation.
- ➢ We know that for a Gaussian random variable, the probability that values of z are in the range ź ± σ is approximately 0.68; the probability is about 0.95 that the values of z are in the range ź ± 2σ.

➢ Rayleigh Noise

➢ The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

➢ The mean and variance of z when this random variable is characterized by a Rayleigh PDF are

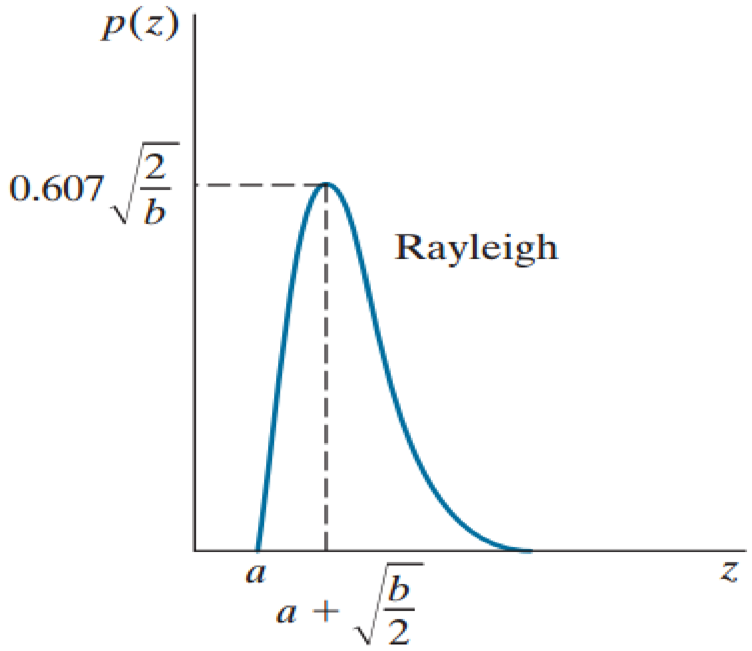
$$\acute{z} = a + \sqrt{\pi\, b/4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

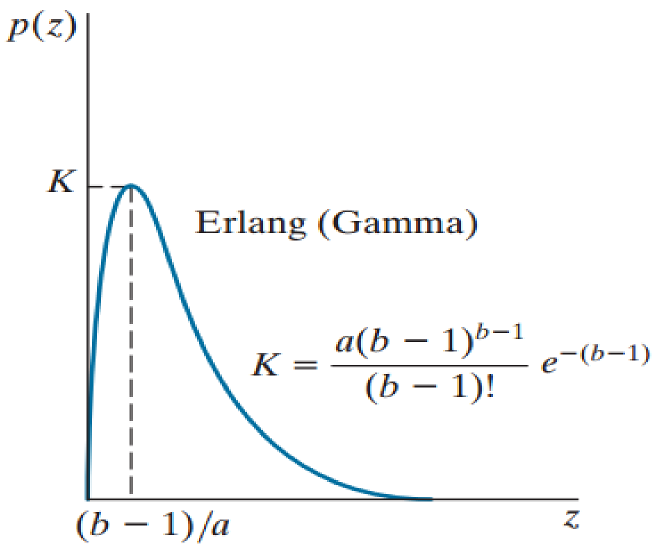
➢ Rayleigh Noise

- Figure (b) shows a plot of the Rayleigh density. Note the displacement from the origin, and the fact that the basic shape of the density is skewed to the right.

- The Rayleigh density can be quite useful for modeling the shape of skewed histograms.



➢ Erlang (Gamma) Noise



➢ The PDF of Erlang noise is

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

➢ where the parameters are such that a>b, b is a positive integer, and "!" indicates factorial. The mean and variance of z are

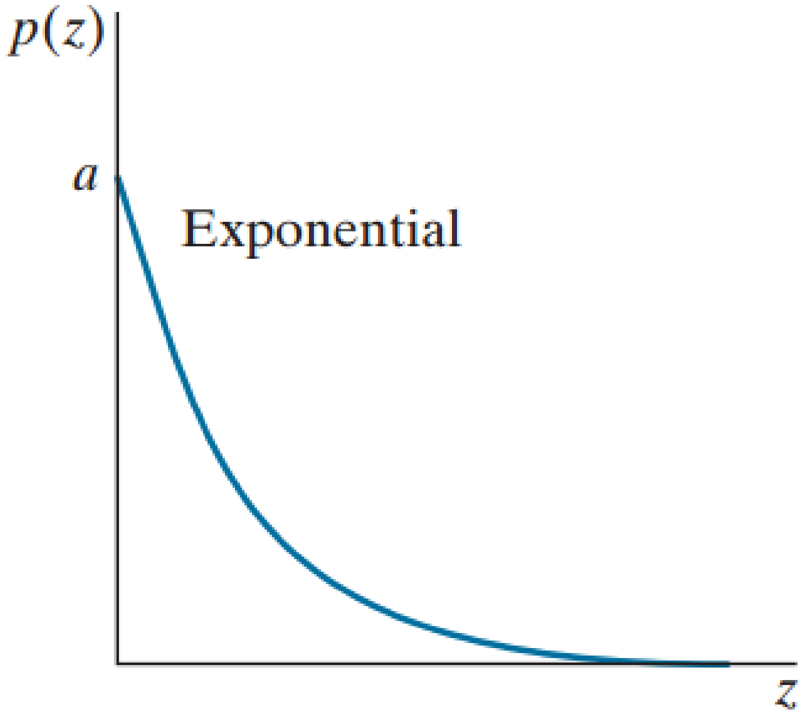
$$\acute{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$

Exponential Noise

- The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$



- where a>b. The mean and variance of z are

$$\acute{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$

➢ Uniform Noise

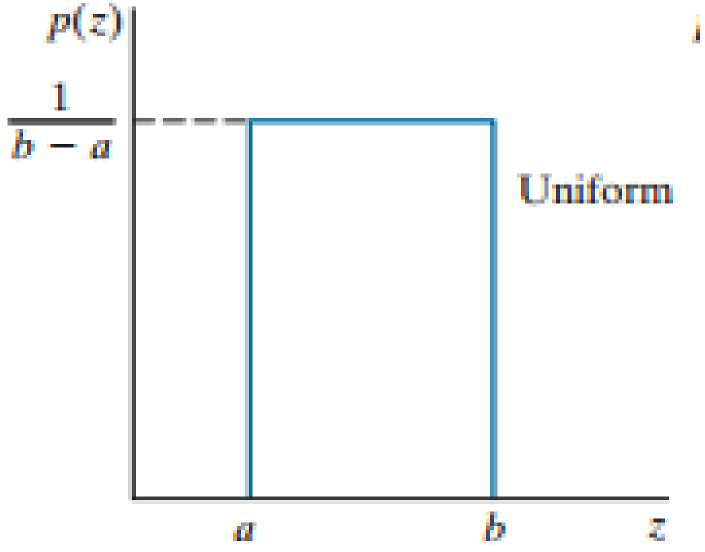
➢ The PDF of uniform noise is

$$p(z) = \begin{cases} \dfrac{1}{b-a} & a \le z \le b \\ 0 & otherwise \end{cases}$$

➢ The mean and variance of z are

$$\acute{z} = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$



➢ Salt-and-Pepper Noise

➢ If k represents the number of bits used to represent the intensity values in a digital image, then the range of possible intensity values for that image is [0.2^k-1] (e.g., [0,255] for an 8-bit image). The PDF of salt-and-pepper noise is given by

$$p(z) = \begin{cases} P_s & for\ z = 2^k - 1 \\ P_p & for\ z = 0 \\ 1 - (P_s + P_p) & for\ z = v \end{cases}$$

where V is any integer value in the range 0 < v < 2^k-1 .


Thank you